

Modelling the Economic Value of Credit Rating Systems

Rainer Jankowitsch ^a, Stefan Pichler ^a, Walter S. A. Schwaiger ^b

^a *Department of Banking Management, Vienna University of Economics and Business Administration, Nordbergstrasse 15, A-1090 Vienna, Austria*

^b *Department of Controlling, Vienna University of Technology, Favoritenstrasse 11, A-1040 Vienna, Austria*

June 2004

Abstract

In this paper we develop a model of the economic value of a credit rating system. Increasing international competition and changes in the regulatory framework driven by the Basel Committee on Banking Supervision (Basel II) called forth incentives for banks to improve their credit rating systems. An improvement of the statistical power of a rating system decreases the potential effects of adverse selection, and, combined with meeting several qualitative standards, decreases the amount of regulatory capital requirements. As a consequence, many banks have to make investment decisions where they have to consider the costs and the potential benefits of improving their rating systems. In our model the quality of a rating system depends on several parameters such as the accuracy of forecasting individual default probabilities and the rating class structure. We measure effects of adverse selection in a competitive one-period framework by parametrizing customer elasticity. Capital requirements are obtained by applying the current framework released by the Basel Committee on Banking Supervision. Results of a numerical analysis indicate that improving a rating system with low accuracy to medium accuracy can increase the annual rate of return on a portfolio by 30 to 40 bp. This effect is even stronger for banks operating in markets with high customer elasticity and high loss rates. Compared to the estimated implementation costs banks could have a strong incentive to invest in their rating systems. The potential of reduced capital requirements on the portfolio return is rather weak compared to the effect of adverse selection.

Key words: Rating system, cohort method, Basel, banking regulation, capital requirements, probability of default, adverse selection.

JEL classification: G28, C13

1 Introduction

Increasing international competition and changes in the regulatory framework driven by the Basel Committee on Banking Supervision (Basel II) called forth incentives for banks to improve their credit rating systems. In a competitive framework a poor statistical power of a bank's internal rating system will deteriorate the economic performance due to adverse selection, i.e. customers with a better credit quality than assessed by the bank will potentially walk away and leave the bank with a portfolio of customers with a credit quality lower than estimated. Obviously, improving the statistical power of a rating system will have a positive impact on economic performance. The size of this effect depends mainly on the degree of competitiveness of the market environment. The counterweight of these potential benefits are the costs of investing into the power of a rating system such as organizational costs, costs of information technology, and costs of collecting and managing the required data. In addition, a bank's internal rating system with sufficient statistical power might be used for calculating the regulatory capital requirements set by the Basel II Internal Ratings Based Approaches which are expected to be lower than in the Modified Standardized Approach. In addition it can be shown that due to the concave relation between regulatory capital requirements and default probabilities even for banks having already qualified for the Internal Ratings Based Approach a more accurate rating system which enables a finer grained rating class structure leads to lower capital requirements.

It is the main objective of this paper to model the decision whether to invest into the quality of a rating system in a rather general framework. Our model is aimed to quantify the benefits of such an investment. The first part of our analysis is focused on the economic value of increasing the statistical power of a bank's internal rating system. In line with the work by Jordão and Stein (2003) we compare the profitability of prototypical banks with different

statistical power of their rating systems in different market environments. In our model the statistical power of a rating system depends on several parameters such as its accuracy and the rating class structure. We measure the accuracy of forecasting individual default probabilities as the variance of the deviations of the forecasted from the true default probabilities. In this setup this measure is more closely related to the economic impact than the area-under-the-curve measures traditionally used by other researchers.

Many banks use cohort based methods to estimate default probabilities rather than individual estimates based on regression models. Since customers of different credit quality are grouped into cohorts and regarded as being of homogeneous credit quality, additional noise may enter the lending decisions. Thus, the numbers of cohorts used by a rating system and the methods to construct their relative sizes (or, put equivalently, the ‘boundaries’ between the cohorts) become additional important parameters which describe the statistical quality of a rating system (for the qualitative standards of state-of-the-art rating systems see, e.g., Krahnen and Weber (2001) and Treacy and Carey (2000)).

When examining the profitability of different prototypical banks we assume that banks adopt a full price-based lending approach rather than a cutoff-based approach. In agreement with the findings of Jordão and Stein (2003) we do not expect any influence on the main results of our analysis by this assumption. The cornerstone of our model is the assumption that a bank possesses estimates (not necessarily free of error) of the true individual default probabilities of all its customers. These estimates may be taken from regression based models which yield individual estimates of default probabilities or from cohort methods where the individual default probabilities are set equal to the average default probability of the cohort. A bank prices the loans offered to its customers according to this estimated default probability. More specifically, the spread over the risk-free rate has to cover the expected loss and the

proportional ‘general’ costs including operating costs and risk premia related to unexpected losses. For simplicity we assume that the ability to measure unexpected losses is not influenced by the statistical power of the rating system. Note that unexpected losses are likely to be very low for large, well diversified portfolios.

We model the competitiveness of the market environment by parametrizing customer elasticity. Customers are assumed to have some better information about their true credit quality. In a full competitive framework with no transaction costs ‘good’ customers who are offered a too high credit spread will eventually walk away to a bank with a more powerful rating system. As a consequence, the bank is left with the ‘bad’ customers who know about their worse credit quality. This adverse selection effect deteriorates the economic performance of the bank and may lead to insolvency of the bank in extreme cases. However, the fraction of ‘good’ customers leaving the bank might not be 100% for several reasons. Firstly, there might be imperfect competition among banks due to oligipolistic structures. Secondly, other banks and/or the customers themselves do not have better risk estimates in all the cases. Finally, there are transaction costs for customers willing to leave the bank which might be prohibitively high. To account for all these possible effects we assume that there is a probability that a customer with a better credit quality does not leave the bank. If this probability is zero we have perfect customer elasticity, if this probability is one there is no competitiveness at all.

Our analysis is restricted to a partial equilibrium framework. As pointed out by Broecker (1990) effects of adverse selection may lead to a situation where only one bank or one rating system exists in a static general equilibrium framework. However, in a dynamic setting the timing of the investment decision – the advantage of being first – is still an interesting

question. For a detailed discussion of preemption in a general dynamic game regarding the adoption of new technologies see Fudenberg and Tirole (1985).

The second part of our analysis is focused on the impact of the statistical quality of a rating system on regulatory capital requirements which are obtained by applying the current proposal released by the Basel Committee on Banking Supervision. Since internal ratings have to fulfill minimum standards regarding their statistical power we consider only cases with a given accuracy of estimating the default probabilities. Based on the concavity of the capital Basel requirement function we deduce that the size of the capital charge decreases with the number of rating classes and, given the number of rating classes, the method to construct the rating cohorts may be of particular importance. Again our framework might be useful determining the economic impact of the rating class structure.

Our model provides a framework to quantify potential positive effects of an improvement of the rating system. Of course in real-world decisions the costs of investing into the power of a rating system have to be taken into account. Considering the fact that due to the advent of the new Basel II regulatory framework most rating systems are in their completion stages it is assumed that banks can make good estimates of their implementation costs. Moreover, these costs can be considered as independent of the decisions of other institutions and thus are readily quantifiable. Earlier estimations for the German market indicated that for the very small banks featuring total assets of up to ten billion Euro the implementation costs would be around one million Euro (one basis point of total assets), while for the middle sized cooperative institutions these would be in the range of five to seven million Euro (Gross et. al., 2002). Subsequently, estimates have been increased to about five basis points of an institution's total assets (Accenture et al. (2004)), whereas current information suggests that even this high figure will most probably be surpassed. We do not extend our analysis,

however, to a formal inclusion of implementation costs. If these costs are known to a bank the decision making process will be obvious. If there is serious uncertainty about the costs then the model will depend heavily on the structure of this uncertainty which is beyond the scope of this paper.

In section 2 we describe the setup of our model. The key ingredients are the distribution of individual default probabilities which captures the portfolio structure, the degree of competitiveness in the market environment, and the way how the accuracy of a rating system is measured. The design of the numerical analysis conducted in this paper is provided in section 3. We described the specific parametrization and the algorithm used to determine the portfolio returns. Section 4 summarizes the numerical results with respect to the adverse selection effect. Section 5 briefly describes the regulatory framework set by the Basel Committee of Banking Supervision and presents the potential effects of reduced capital requirements. Section 6 concludes the paper.

2 Model Setup

In this section we describe the setup for evaluating the economic impact of rating systems with different predictive power. Many banks are expected to base their PD estimation on the observation of empirical default rates within rating classes. This so-called "cohort method" (see eg Jarrow et al. (1997) and Lando and Skodeberg (2002)) is the basic object of our analysis. The main alternative, however, the usage of regression-based forecasts of individual PDs can be seen as a special case of our framework where we have one customer per rating class or - put more precisely - one rating class per credit or regression score because it is possible to observe customers with identical credit scores or regression outputs. In this section

we will describe a model to quantify the effect of adverse selection which can be used to indicate if there is an incentive for banks to invest in the improvement of their credit rating systems.

In our setup the credit portfolio of a bank is characterized by the number of customers and the “true” probability of default of each customer. We assume that the recovery rates are known for all customers. So we concentrate on the quality of rating systems with respect to the estimation of PDs. Of course, improving the predictive power of the estimation of recovery rates will also have a significant economic value but we leave this topic open for future research. To simplify the analysis we assume that all exposures are of equal size, which is a reasonable approximation for a large, well diversified portfolio, and that the PDs in the portfolio may be described by a certain ex-ante distribution, which describes the PD distribution of all potential customers for the bank. In our numerical approach the true PD for each customer in the portfolio is drawn from this distribution.

The rating system of the bank will only provide estimates for the true PD of each customer. The difference between the estimated and the true PD will depend on the number and sizes of the rating classes, and the measurement error of the PD. The number of rating classes can be freely chosen by the bank. We will assume that the bank estimates the credit score of each customer and uses this information to slot the customer into a particular rating class. In line with logistic regression models we assume that the relation between credit scores and PDs is

$$PD = \frac{1}{1 + e^{-\text{credit score}}} \quad (1)$$

The bank has to choose the PD-boundaries or credit score boundaries to distribute the customers among the rating classes. There is no natural optimal solution to this problem. Different banks will divide up their customers according to different rules, e.g. approximately equal number of customers per rating cohort. In the case of infinitely many rating classes this slotting is not necessary. This method is equivalent with using the PDs directly from the regression model.

Once the customers are slotted into rating classes the bank estimates the PD of each rating class and uses this PD for pricing and risk management of the customers of this rating class. The estimated PD of each rating class is taken as the expected number of defaults divided by the number of customers. This is the typical procedure for calculating PDs for rating classes and is also applied by rating agencies like Standard&Poor's or Moody's to provide PDs related to their rating classes.

If we assume no measurement errors the choice of only one rating class will always maximise the difference over all individual customers between estimated and true PD. Provided that individual PDs differ across customers the deviations of individual PDs from the overall average PD of the portfolio (which equals the PD of the single rating class) are obviously higher than deviations from subgroup averages. Thus without measurement errors the bank can reduce the difference between estimated and true PD by using more and dispersed rating classes.

In the next step we introduce measurement errors for the estimated credit score. We assume that the errors are normally distributed with mean zero:

$$credit\ score_{estimated} = credit\ score_{true} + \varepsilon \quad (2)$$

$$\varepsilon \sim N(0, \sigma^2) \quad (3)$$

The parameter σ controls the magnitude of the estimation error. Introducing a measurement error means that customers are potentially slotted into the wrong rating class and therefore being stronger exposed to adverse selection than in the situation without measurement error. Figure 1 shows the effect of measurement errors for low, medium, high, and perfect accuracy in estimating the credit scores. For each level of accuracy the one standard deviation confidence level is plotted:

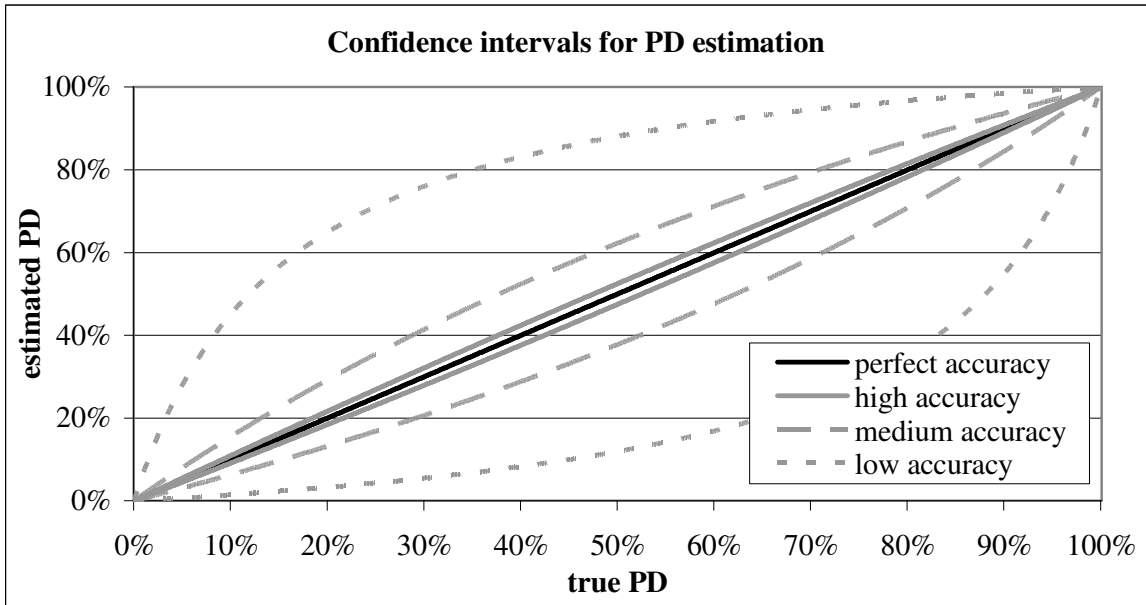


Figure 1: One-standard deviation confidence intervals for PD estimation for perfect ($\sigma = 0$), high ($\sigma = 0.1$), medium ($\sigma = 0.5$) and low ($\sigma = 2$) accuracy.

The number and sizes of rating classes and the parameter σ of the measurement error are under the control of the bank. Investing into the predictive power of a rating system thus means to be able to reduce the measurement error and to use more und better dispersed rating classes. The first goal of this paper is to analyse the effect of these parameters on the return of different portfolios and in different market situations to obtain the optimal strategy concerning the investment into the rating system.

Given a certain portfolio which is characterised by the number of customers and by a certain ex-ante distribution for the true PDs and given the predictive power of a rating system characterised by the number und sizes of the rating classes and by the measurement error σ we analyse the return of certain portfolios.

Since we want to apply a risk-adjusted pricing we first define the pricing mechanism. We assume that the bank needs to receive a certain interest rate r to cover all ‘general’ costs besides credit risk related to the expected loss. The general costs include operating costs and risk premia related to unexpected losses. We assume that the ability to measure unexpected losses is not influenced by the decisions in our model. Credit risk related to expected losses is priced by demanding a certain credit spread s . For simplicity we restrict our analysis to one-period zero-coupon bonds. The credit spread depends on the PD and the loss-given-default (LGD) of the individual exposures. If no default occurs the bank receives $(1+r+s)$, if default occurs the bank receives $(1+r+s) \cdot (1-LGD)$. In assuming the LGD to be constant in line with the assumption of the foundation IRB approach the credit spread is a function of PD, LGD, and r . In essence, the expected payoff of the loan has to be equal to the desired value:

$$1 + r = (1 - PD) \cdot (1 + r + s) + PD \cdot (1 - LGD) \cdot (1 + r + s) \quad (4)$$

Solving for s :

$$s = (1 + r) \cdot \frac{PD \cdot LGD}{1 - PD \cdot LGD} \quad (5)$$

Using this pricing mechanism we are now able to introduce our concept of adverse selection. The rating system of a bank provides the estimated PD for each customer who applies for a loan. Using this PD and an estimate for the LGD the credit spread which is offered to customers can be calculated by the pricing mechanism. If the PD is overestimated the customer will be offered a credit spread, which is too high compared to her true PD. We will assume that customers, who are offered a too high spread, will leave the bank with a certain probability which is dependent on the magnitude m of the deviation from the fair spread.

$$m = s_{estimated} - s_{fair} = (1 + r) \cdot \frac{PD_{estimated} \cdot LGD}{1 - PD_{estimated} \cdot LGD} - (1 + r) \cdot \frac{PD_{true} \cdot LGD}{1 - PD_{true} \cdot LGD} \quad (6)$$

There are several possible reasons why a customer might not leave the bank in a situation where she is offered a too high spread. First of all, one can imagine that a customer is not better informed about her default probability than the bank. A second reason could be related to the degree of competitiveness in the market. It is possible that other banks do not have better rating systems as well and do not offer more attractive spreads to an informed customer who is willing to leave her bank. Finally, transaction costs and cross selling effects may also serve as an additional reason for customers to pay higher spreads to their bank. Since we do not want to separate among these effects it suffices to model the outcome of the customers' decisions using a simple probability distribution. The probability to leave the bank is dependent on m and on the elasticity of the customer. To model the elasticity of the customer

we assume the following functional relation between the probability of leaving the bank and the estimation error in the spread

$$probability\ to\ leave = 1 - e^{-\alpha \cdot m} \quad (7)$$

where α is a elasticity parameter. If α is zero, all customers will stay. If α goes to infinity, all customers with an overestimated PD will leave the bank. All customers who are offered a fair or a lower spread will stay with the bank. Table 2 shows the probability to leave the bank for a low, medium, and high level of elasticity. Note that given empirical observations of percentages like in table 2 one could calibrate the free parameter α to real-world data.

fair spread	elasticity		
	low	medium	high
+ 5 bp	4.9 %	22.1%	99.3%
+ 10 bp	9.5%	39.3%	99.9%
+ 50 bp	39.3%	91.8%	100.0%

Table 2: Probability to leave the bank for certain levels of elasticity. Probabilities are presented for different deviations of the offered spread from the fair spread (left column) and for low ($\alpha = 100$), medium ($\alpha = 500$), and high ($\alpha = 10,000$) elasticity, respectively.

In the simulation approach this probability to leave the bank is used to quantify the effect of adverse selection. We will analyse the effects using different degrees of elasticity. If the bank knows the true probability of default of each customer it will on average earn an interest rate of r on the portfolio. If the estimated and true PD differs the return on the portfolio will be lower than r dependent on the elasticity of the customers, which generates the effect of adverse selection.

3 Design of numerical analysis

In this section we briefly explain the simulation approach based on the model setup described in the previous chapter. First of all we want to quantify the effect of adverse selection for different portfolios. A portfolio is described by the number of customers and their individual “true” probability of default. In the numerical analysis we fix the number of customers at 10,000. This number represents a well diversified portfolio implying that the observed loss is likely to be close to the expected loss. After fixing the number of customers we have to determine their true PDs. As explained in section 2 the true PD for each customer in the portfolio is drawn from a certain ex-ante distribution, which describes the PD distribution of all potential customers for the bank. We choose to use the Beta distribution for this analysis because it is easy to handle and has some attractive properties, e.g. it is defined over a finite interval as PDs are and it allows for extreme skewness as we expect for PD distributions (see Renault and Scaillet (2003) on the use of Beta distributions to model recovery rates). To compare the effects of adverse selection for portfolios of different quality three Beta distributions have been chosen, from which the PDs are drawn:

Good portfolio: Beta distribution with $p = 0.4$ and $q = 19$ (median PD = 0.77%)

These parameters are chosen such that the distribution represents a quite good corporate portfolio. Compared to the average portfolio this means that we observe more customers with very small PDs.

Average portfolio: Beta distribution with $p = 0.7$ and $q = 37.6$ (median PD = 1.08%)

These parameters are estimated by using a dataset of more than 30,000 Austrian corporate customers provided by Creditreform (see Schwaiger (2003)). Employing a logistic regression we estimate individual PDs for each customer in the dataset. We use a maximum likelihood estimation to find the parameters for the Beta distribution which optimally explains the individual PDs. Thus, this distribution represents a real portfolio.

Weak portfolio: Beta distribution with $p = 1.4$ and $q = 58$ (median PD = 1.84%)

These parameters are chosen such that the distribution represents a rather weak corporate portfolio. Compared to the distribution of the average portfolio this means that we observe more customers with high PDs.

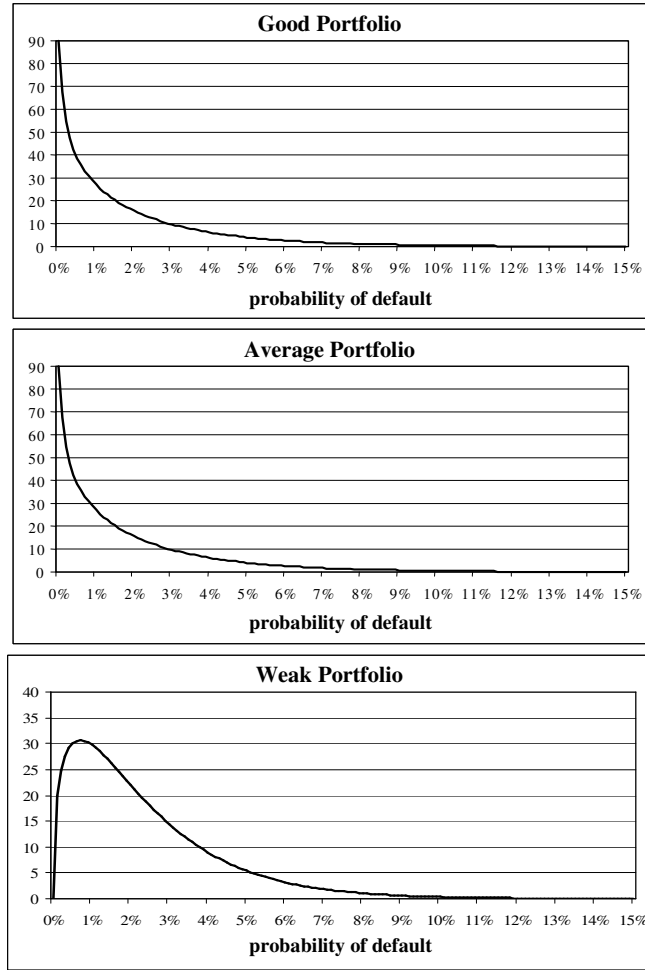


Figure 2: Densities of the Beta distributions representing a good ($p = 0.4$ and $q = 19$), an average ($p = 0.7$ and $q = 37.6$), and a bad ($p = 1.4$ and $q = 58$) portfolio. The parameters of the average portfolio are calibrated to a dataset of more than 30,000 Austrian corporate customers provided by Creditreform (see Schwaiger (2003)).

Given the number of customers and their true PDs we generate the credit scores the bank observes. This is achieved by transforming the PD drawn from the relevant Beta distribution to a true credit score using equation (1) and by adding a simulated measurement error for each customer as described in equation (2). The observed credit scores are dependent on the measurement errors of the rating system. The measurement error is controlled by the parameter σ . For the numerical analysis we will present results for low ($\sigma = 2$), medium ($\sigma = 0.5$), high ($\sigma = 0.1$), and perfect ($\sigma = 0$) accuracy in estimating the credit scores. Given data of estimated credit scores and actual defaults (as required by the Basel IRB framework) values for σ can easily be fitted to empirical data.

In the next step the bank has to choose the number and sizes of the rating classes. We will consider banks which use one, two, five, ten, and infinitely many rating classes. Concerning the sizes of the rating classes there is no natural best solution to define credit score boundaries. For the numerical analysis we propose four different methods, which seem to be reasonable ways in defining the sizes of rating classes (see appendix).

After slotting the customers in rating classes by using the observed credit spreads the bank estimates the PD of each class. This PD is taken as the expected number of defaults divided by the number of customers.

The next parameter, which is necessary for the bank to price loans, is the LGD. We will assume that the LGD is equal for all customers and known to the bank. The effect of the LGD estimation is an interesting topic, but in this paper we want to focus on the effects of the PD estimation. Nevertheless we will observe the effects of adverse selection for three different LGD-levels: low (25%), medium (45% as proposed by the Basel II IRB foundation approach), and high (75%).

Having an estimate for PD and LGD for each customer the bank can calculate the credit spread for each customer by equation (5) given some interest rate r , which covers all costs besides credit risk related to the expected loss. We set this interest rate r to 3%, but the results are virtually the same for any other reasonable level of r .

The customers decide then whether to accept or to reject the loan offered to them. Every customer, who is offered the fair or a lower credit spread, will accept the loan. All customers offered a too high credit spread will reject the loan with a certain probability depending on the

elasticity parameter α . To model different behaviour of customers we choose three levels of elasticity: low ($\alpha = 100$), medium ($\alpha = 500$), and high ($\alpha = 10,000$).

Given the level of elasticity we simulate which customers leave the portfolio and which stay. Through adverse selection even with a low level of elasticity the bank will loose some customers. After determining which customers form the actual portfolio of the bank, we are now ready to calculate the return of this portfolio $r_{portfolio}$ which is the average over the returns on the individual loans r_i . To calculate r_i we simulate which customers in the portfolio actually default using their individual true PDs:

$$1 + r_i = \begin{cases} 1 + r + s & \text{no default of customer } i \\ (1 + r + s) \cdot (1 - LGD) & \text{default of customer } i \end{cases} \quad (8)$$

$$r_{portfolio} = \frac{1}{n} \sum_{i=1}^n r_i \quad (9)$$

The return of the portfolio is the result of one simulation path. For each combination of the parameters we run 100 simulations to estimate the average of the portfolio returns. These average returns are the main results of our numerical analysis. We are able to examine the portfolio return effects for different parameter constellations. The main task to compare rating systems with different predictive power can be achieved by simulating their returns in the proposed way.

4 Numerical results

In this section we quantify the effect of rating systems with different predictive power on the portfolio return. Investing into a better rating system means to use more and better dispersed rating cohorts and to reduce the measurement error in the estimation of the credit scores. In the first analysis we want to concentrate just on the effect of improving the measurement error. To do this we define one base case:

Base Case: number of rating cohorts: 10
 sizes of the rating cohorts: linearly increasing number of defaults (method 4)
 number of customers: 10,000
 LGD: medium (45%)
 elasticity: medium ($\alpha = 500$)

For this base case we quantify the effect on the portfolio return when improving the accuracy of the PD estimation for the three different portfolios represented by Beta distributions (see section 3). Table 3 shows the increase of the portfolio return when improving a rating system with low accuracy to medium, high or perfect accuracy:

base case	accuracy of PD estimation		
	medium	high	perfect
good portfolio	30.8	43.7	44.8
average portfolio	32.6	45.9	46.8
weak portfolio	39.0	56.4	58.7

Table 3: Increase in portfolio return (in bp) for the base case when improving the accuracy of the rating system from a low accuracy level ($\sigma=2$) to a medium ($\sigma=0.5$), high ($\sigma=0.1$), and perfect ($\sigma=0$) accuracy.

For realistic portfolios the return increases by 30 to 40 bp p.a. when the bank upgrades its rating system from low to medium accuracy. Improving from medium to high accuracy increases the portfolio return still by approximately 15 bp. Moving from high to perfect accuracy results only in an approximate 1 bp improvement.

These results indicate that an improvement of the rating system has a very strong effect for banks with low or medium accuracy systems. Avoiding the effect of adverse selection improves the portfolio return significantly. The effect is also stronger for banks with rather weak portfolios.

In the next step we present results of the portfolio return where the parameter values of our base case are changed. We want to analyse if banks with certain characteristic in their portfolio (e.g. highly collateralised loans) have more incentives to invest in their rating system.

In order to check for the influence of the degree of competitiveness in the market environment we change the elasticity parameter of the base case. One would expect that for banks with more elastic customers, i.e. who leave the bank with higher probability if they are offered a too high credit spread, the improvement of the rating system is more important. Table 4 and 5 show the increase of the portfolio return when improving a rating system with low accuracy to medium, high, or perfect accuracy in the case of high and low elasticity:

high elasticity	accuracy of PD estimation		
	medium	high	perfect
good portfolio	32.4	49.8	53.7
average portfolio	34.2	51.6	55.8
weak portfolio	36.1	58.9	62.9

Table 4: Increase in portfolio return (in bp) given high customer elasticity ($\alpha=10,000$) when improving the accuracy of the rating system from a low accuracy level ($\sigma=2$) to a medium ($\sigma=0.5$), high ($\sigma=0.1$), and perfect ($\sigma=0$) accuracy.

low elasticity	accuracy of PD estimation		
	medium	high	perfect
good portfolio	18.6	25.3	26.1
average portfolio	19.7	26.4	27.3
weak portfolio	25.2	32.7	33.8

Table 5: Increase in portfolio return (in bp) for a low customer elasticity ($\alpha=100$) when improving the accuracy of the rating system from a low accuracy level ($\sigma=2$) to a medium ($\sigma=0.5$), high ($\sigma=0.1$), and perfect ($\sigma=0$) accuracy.

The results clearly show that in loan markets with higher customer elasticity the improvement is significantly stronger. In markets with oligopolistic structures and high market power for a bank the adverse selection effect is not that important but still around 20 bp.

Next we analyse the effect of the LGD on the improvement potential of the rating system. We compare portfolios with high LGD (75%) and low LGD (25%). We would expect that the improvement of the rating system is more important for portfolios with high LGD. Table 6 and 7 show the increase of the portfolio return when improving a rating system with low accuracy to medium, high or perfect accuracy in the case of high and low LGD:

high LGD	accuracy of PD estimation		
	medium	high	perfect
good portfolio	53.9	78.9	84.0
average portfolio	55.0	80.8	84.6
weak portfolio	62.0	96.8	102.3

Table 6: Increase in portfolio return (in bp) for a high LGD (75%) when improving the accuracy of the rating system from a low accuracy level ($\sigma=2$) to a medium ($\sigma=0.5$), high ($\sigma=0.1$), and perfect ($\sigma=0$) accuracy.

low LGD	accuracy of PD estimation		
	medium	high	Perfect
good portfolio	16.3	21.8	22.1
average portfolio	16.9	22.8	22.9
weak portfolio	21.6	29.4	29.6

Table 7: Increase in portfolio return (in bp) for a low LGD (25%) when improving the accuracy of the rating system from a low accuracy level ($\sigma=2$) to a medium ($\sigma=0.5$), high ($\sigma=0.1$), and perfect ($\sigma=0$) accuracy.

The results indicate that the LGD is very important for the size of the effect of improving rating accuracy. Banks with low LGD, e.g. due to highly collateralised loans, do not depend that much on the quality of their PD estimation. On the other side banks with completely uncollateralised loans depend heavily on the accuracy of their PD estimation.

In the last analysis we compare the effect of rating accuracy for a different number of rating cohorts. In the base case we used ten rating cohorts. Now we use five and infinitely many ratings cohorts to analyse the effect.

five rating cohorts	accuracy of PD estimation		
	medium	high	Perfect
good portfolio	28.6	40.5	40.8
average portfolio	29.7	41.4	41.6
weak portfolio	34.9	50.2	50.6

Table 8: Increase in portfolio return (in bp) for five rating cohorts when improving the accuracy of the rating system from a low accuracy level ($\sigma=2$) to a medium ($\sigma=0.5$), high ($\sigma=0.1$), and perfect ($\sigma=0$) accuracy.

infinitely many rating cohorts	accuracy of PD estimation		
	medium	high	Perfect
good portfolio	32.2	46.8	47.7
average portfolio	34.3	47.9	49.4
weak portfolio	41.8	60.2	63.3

Table 9: Increase in portfolio return (in bp) for infinitely many rating cohorts when improving the accuracy of the rating system from a low accuracy level ($\sigma=2$) to a medium ($\sigma=0.5$), high ($\sigma=0.1$), and perfect ($\sigma=0$) accuracy.

As expected, the effect of rating accuracy is more important for banks that use more rating cohorts. Banks that use a low number of rating cohorts have only a rough measure for the individual PDs of the customers even if they can measure the credit score without error. Thus improving rating accuracy is not that important for their situation.

5 Capital requirements

In this section we analyse the effect of improving the rating system on the regulatory capital requirements of a bank. The Basel Committee on Banking Supervision previously has released a series of consultative documents, accompanying working papers, and finally the new capital adequacy framework commonly known as Basel II. One of the cornerstones of this new Capital Accord ("Pillar 1") is a new risk-sensitive regulatory framework for a bank's

own calculation of regulatory capital for its credit portfolio. Banks which qualify themselves in terms of data availability, statistical methods, risk management capabilities, and a number of additional qualitative requirements will be allowed to adopt the Internal Rating Based (IRB) approach to calculate their capital requirements. In the Foundation IRB (FIRB) approach banks can use their own PD estimates of their customers. In the Advanced IRB approach they can use own estimates of average loss rates and credit conversion factors additionally.

We do not focus on institutional changes in regulatory capital related to differences in the formulas for the risk weighted capital in the Modified Standardized Approach and the FIRB. We concentrate rather on the economic value of improving a rating system given that a bank has already qualified itself for the FIRB approach.

In the FIRB approach banks are allowed to use internal PD estimates to calculate the effect on capital requirements for different rating systems using the proposed formulas suggested by the Basel Committee. For expository purposes we concentrate on the formula for corporate customers but the essence of our results will hold for other customer classes (retail, banks, sovereigns) as well. The proposed capital requirement (CR) of a standard uncollateralized corporate exposure expressed as a function of the customer's PD consists of two parts. The first part represents the capital requirement for the unexpected loss (CR_UL):

$$CR_UL(PD) = \left[LGD \cdot N \left(\sqrt{\frac{I}{1-R(PD)}} \cdot G(PD) + \sqrt{\frac{R(PD)}{1-R(PD)}} \cdot G(0.999) \right) - PD \cdot LGD \right] \cdot \frac{I}{1-1.5 \cdot b(PD)} \cdot (1 + (M - 2.5) \cdot b(PD)) \quad (10)$$

with

$$R(PD) = 0.12 \cdot \frac{1 - e^{-50 \cdot PD}}{1 - e^{-50}} + 0.24 \cdot \left(1 - \frac{1 - e^{-50 \cdot PD}}{1 - e^{-50}} \right) \quad (11)$$

$$b(PD) = (0.11852 - 0.05478 \cdot \log(PD))^2 \quad (12)$$

where $N(.)$ Standard normal cumulative distribution function

$G(.)$ Standard normal inverse cumulative distribution function

LGD Loss-given-default; in the FIRB the LGD is set equal to 45% for standard uncollateralized corporate exposures

PD For corporate exposures we have $PD = \max(PD^*; 0.03\%)$, where PD^* denotes the estimated PD of the customer

M Effective maturity; in the FIRB the effective maturity is set equal to 2.5 years.

The second part represents the capital requirement for the expected loss (CR_EL):

$$CR_EL(PD) = PD \cdot LGD - \text{total eligible provisions} \quad (13)$$

For our analysis we set the total eligible provisions to zero. As long as the provisions of customers are equal this assumption does not affect the results at all. The capital requirement for a customer is then the sum of the capital requirement for expected and unexpected loss:

$$CR(PD) = CR_EL(PD) + CR_UL(PD) \quad (14)$$

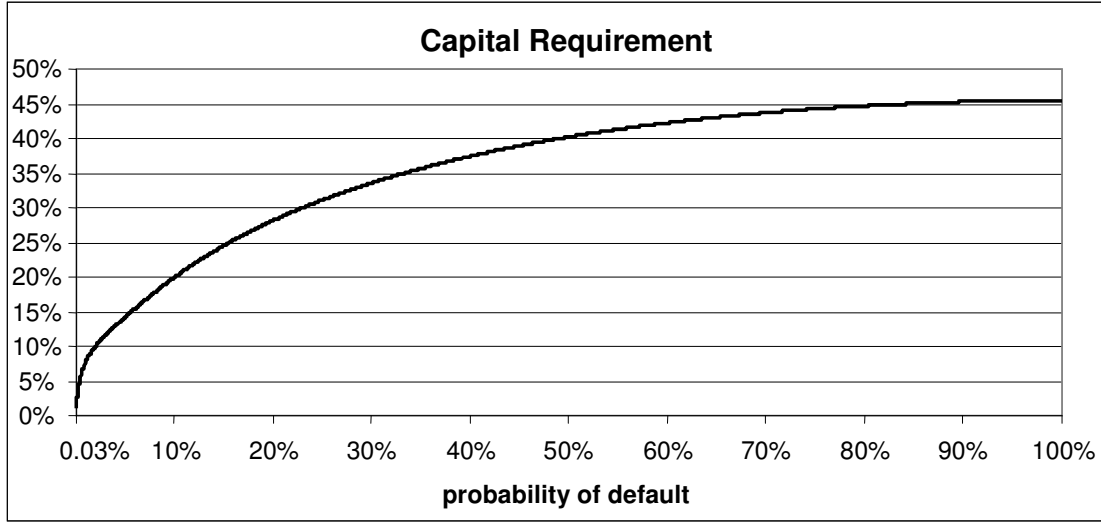


Figure 3: Capital requirement function for standard uncollateralized corporate exposures.

For the capital requirement the PDs of the customers have to be estimated. We will use PDs which are measured without error to take into account the quality standards under the capital accord. Thus the accuracy of a rating system is represented by the number and sizes of the rating classes. It is our objective to measure the effect of these two parameters on the capital requirement of certain portfolios. An investment in the rating system means to be able to divide up the portfolio into more and dispersed rating classes according to the quality standards of Basel II.

Consider a rating class defined by an arbitrary PD or credit score interval. The capital requirement for this rating class is obtained by

$$CR(E[PD_i, i \in \text{rating class}]) \quad (15)$$

where $CR(\cdot)$ denotes the capital requirement function and $E[PD_i, i \in \text{rating class}]$ denotes the expected (or average) PD of rating class i . If the rating class i is divided into two non-empty subclasses the capital requirement is given by

$$CR(E[PD_j, j \in \text{rating subclass 1}]) + CR(E[PD_k, k \in \text{rating subclass 2}]) \quad (16)$$

The function to calculate the capital requirements out of the PDs, which is given in (10-12), is strictly concave for PD values greater than the “floor” PD of 3 bp. As a consequence, it follows from Jensen's inequality that

$$CR(E[PD_i, i \in \text{rating class}]) > CR(E[PD_j, j \in \text{subclass 1}]) + CR(E[PD_k, k \in \text{subclass 2}]) \quad (17)$$

Thus, having an iterative application of this argument in mind we conclude that the finer the rating system the lower the regulatory capital requirement. In figure 4 we show the difference in the capital requirements when we have two customers (A and B) and the PD is firstly estimated for each customer and then the PD is estimated for the portfolio of the two customers.

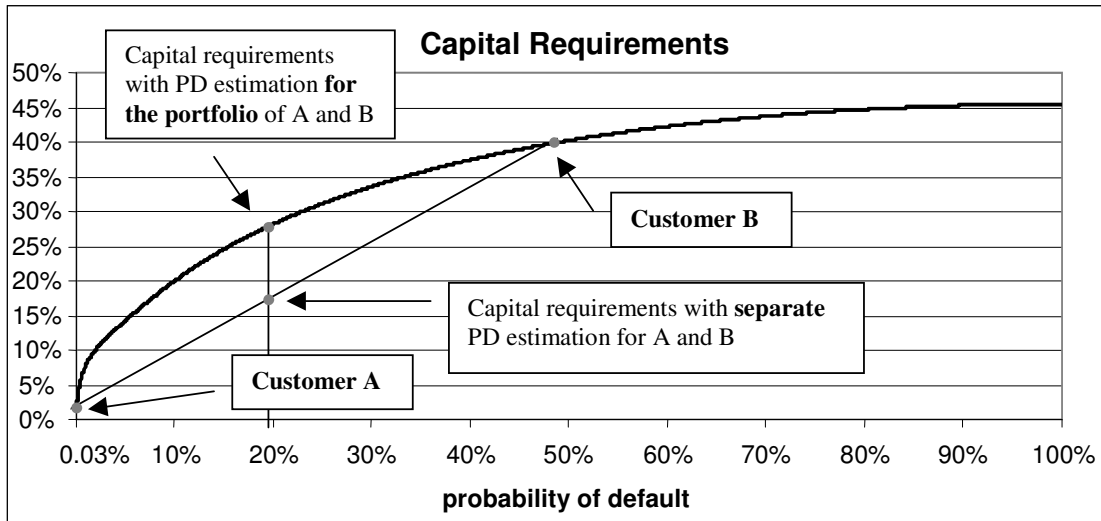


Figure 4: Consequences of Jensen's inequality on the calculation of the capital requirements.

From the theoretical point of view we can deduct that the structure of a rating system has a potential impact on a financial institution's capital requirements. The main result based on Jensen's inequality is that the finer the rating system, the lower the capital requirements. However, we cannot deduct any indication about the potential size of these effects. To provide such quantification we make more specific assumptions about the distribution of PDs using the three different portfolios described in section 3.

Given the density of the Beta distribution for all potential customers we can infer how many percent of the customers in the portfolio have a PD inside the interval $[x_1, x_2]$ and we can obtain the expected PD for this group of customers analytically. This is all we need to know to calculate the capital requirement for a set of rating cohorts where the corresponding cohort boundaries are given by their minimal and the maximal PDs.

The expected value for the PD of a customer given that the customer has a PD between x_1 and x_2 has the following functional form

$$E[PD | x_1 \leq PD \leq x_2] = \frac{p}{p+q} \cdot \frac{cdf(p+1, q, x_2) - cdf(p+1, q, x_1)}{cdf(p, q, x_2) - cdf(p, q, x_1)} \quad (18)$$

where $cdf(.)$ denotes the cumulative density function of a Beta distribution with parameters p and q and $B(.,.)$ denotes the Beta function. Provided this analytical relation we can avoid a simulation procedure compared to section 4.

We calculate the capital requirement for the three portfolios under consideration using one, two, five, ten, and infinitely many rating cohorts and using the four different methods to construct credit score intervals (see appendix). Applying these methods of defining the sizes

of the rating classes the capital requirement for a different number of rating classes can be compared. Tables 10 to 12 show the resulting capital requirements for the four methods over the different Beta distributions of our three portfolios:

good portfolio	1 cohort	2 cohorts	5 cohorts	10 cohorts	∞ cohorts	reduced capital requirements using 10 instead of 5 cohorts
method 1	9.56%	9.55%	9.29%	8.84%	7.30%	45 bp
method 2	9.56%	7.91%	7.47%	7.36%	7.30%	11 bp
method 3	9.56%	8.94%	8.22%	7.87%	7.30%	35 bp
method 4	9.56%	8.63%	7.75%	7.48%	7.30%	27 bp

Table 10: Capital requirements for the good portfolio using one, two, five, ten, and infinitely many rating cohorts for different methods of defining the sizes of the rating cohorts.

average portfolio	1 cohort	2 cohorts	5 cohorts	10 cohorts	∞ cohorts	reduced capital requirements using 10 instead of 5 cohorts
method 1	9.37%	9.37%	9.25%	8.93%	8.00%	32 bp
method 2	9.37%	8.43%	8.11%	8.04%	8.00%	7 bp
method 3	9.37%	8.90%	8.46%	8.27%	8.00%	19 bp
method 4	9.37%	8.69%	8.21%	8.08%	8.00%	13 bp

Table 11: Capital requirements for the average portfolio using one, two, five, ten, and infinitely many rating cohorts for different methods of defining the sizes of the rating cohorts.

weak portfolio	1 cohort	2 cohorts	5 cohorts	10 cohorts	∞ cohorts	reduced capital requirements using 10 instead of 5 cohorts
method 1	9.99%	9.99%	9.95%	9.76%	9.25%	19 bp
method 2	9.99%	9.52%	9.33%	9.28%	9.25%	5 bp
method 3	9.99%	9.71%	9.48%	9.37%	9.25%	11 bp
method 4	9.99%	9.60%	9.36%	9.28%	9.25%	8 bp

Table 12: Capital requirements for the weak portfolio using one, two, five, ten, and infinitely many rating cohorts for different methods of defining the sizes of the rating cohorts.

Using more and better dispersed rating classes a bank can save a significant amount of regulatory capital. Depending on the portfolio and on the method of defining the sizes of the rating classes the capital requirements can be reduced by up to 45 bp when using ten instead

of five rating classes. On average a bank which increases the number of rating classes from five to ten can expect a lower capital requirement of around 10 to 20 bp. Increasing the number of rating classes from ten to infinity can still be important for certain methods of defining the rating classes. However for more sophisticated methods (e.g. method 2) the effect is comparably small (around 3 bp). Analysing the results of the capital requirement for the different methods of defining the sizes of rating classes we conclude that method 2 (equal number of customers per rating class) and method 4 (linearly increasing number of expected defaults per rating class) are the most promising concepts. The magnitude of the differences between these two methods is rather small. Method 1 (equally spaced PD intervals) and method 3 (equal number of expected defaults) are suboptimal solutions.

In order to compare the magnitude of the economic value of reduced capital requirements one has to convert these figures into annual returns. The key question is how to determine the appropriate costs of capital. First of all, we assume that there is no free regulatory capital because in such situations the economic value of reduced capital requirements diminishes. Since this study is aimed to analyse potential effects of improving the quality of rating systems it is sufficient to quantify the effects for a range of reasonable inputs. Thus for rather low costs of capital, say 3%, a 20 bp reduction of regulatory capital translates into a 0.6 bp increase of the annual rate of return. Even in the case of high costs of capital, say 10%, this effect is only 2 bp. Compared to the effect of adverse selection the potential reduction in the capital requirements is much lower. We conclude that incentives provided by the Basel regulatory framework might not suffice to cover the costs of improving a rating system in many cases given the high estimates of implementation costs (see Accenture et al. (2004), Gross et al. (2002)). However, in combination with the adverse selection effect the benefits of an investment into the quality of a rating system have the potential to outweigh the costs.

6 Conclusion

In this paper we develop a model to determine the potential economic value of improving a credit rating system. We describe a credit rating system by the number and sizes of the rating classes and the measurement error in the estimation of individual default probabilities. Our model is aimed to advise banks when making an investment decision with respect to the quality of their rating systems. All model parameters are designed to be empirically observable or at least to be calibrated to empirically observable values.

In a first step we analyze the potential effect of adverse selection on credit portfolio return for rating systems with different accuracy. Our findings indicate that improving a rating system with low accuracy to medium accuracy can increase the annual rate of return by 30 to 40 bp. This effect is even stronger for banks operating in markets with high customer elasticity and high loss rates. Compared to the estimated implementation costs banks could have a strong incentive to invest in their rating system.

In a second step we analyse the effects of the reduction of regulatory capital requirements under the Basel FIRB approach. Improving the accuracy of rating systems gives the bank the opportunity to make use of a finer grained rating system, i.e. to use a higher number of rating classes. The concavity of the regulatory capital formula implies that capital requirements are lower for finer grained rating systems. Our results show that the potential of this effect on the portfolio return is rather weak compared to the effect of adverse selection.

References

Accenture, Mercer Oliver Wyman, and SAP, 2004, Reality Check on Basel II, Special Supplement to The Banker, July 2004, 152-161.

Basel Committee on Banking Supervision, 2004, International Convergence of Capital Measurement and Capital Standards: a Revised Framework, Basel, June 2004, www.bis.org.

Broecker, T., 1990, Credit-Worthiness Tests and Interbank Competition, *Econometrica* 58/2, 429-452.

Fudenberg, D., Tirole, J., 1985, Preemption and Rent Equalization in the Adoption of a New Technology, *Review of Economic Studies* 52/3, 383-401.

Gross, T., Kolbeck, C., Nicolai, A., Rockenfelt, M., 2002, Deutsche Banken auf dem Weg zu Basel II, Research Report, Boston Consulting Group and University of Witten/Herdecke.

Jarrow, R., Lando, D., Turnbull, S., 1997, A Markov model for the term structure of credit risk spreads, *Review of Financial Studies* 10 (2), 481 – 523.

Jordão, F., Stein, R., 2003, What is a more powerful model worth? Moody's – KMV Technical report #030124.

Krahnen, J.P., Weber, M., 2001, Generally accepted rating principles: A primer, *Journal of Banking & Finance*, 25 (1), 3 – 23.

Lando, D., Skodeberg, T., 2002, Analyzing rating transitions and rating drift with continuous observations, *Journal of Banking & Finance*, 26 (2/3), 423 – 444.

Renault, O., Scaillet, O., 2003, On the way to recovery: A nonparametric bias free estimation of recovery rate densities, FAME Research Paper No. 83, HEC Geneve.

Schwaiger, W., 2003, Basle II: Quantitative impact study on small and medium-sized enterprises in Austria, in: Chen et al. (Editors): *The Role of International Institutions in Globalisation: The Challenges of Reform*, Edgar Elgar, pp. 166 – 180.

Treacy, W. F., Carey, M., 2000, Credit rating systems at large US banks, *Journal of Banking & Finance*, 24 (1/2), 167 – 201.

Appendix

Concerning the sizes of the rating classes there is no natural best solution to define credit score boundaries. For the numerical analysis we propose four different methods, which seem to be reasonable ways in defining the sizes of rating classes:

Method 1: The maximal PD for the worst customer is taken and divided by the numbers of cohorts. The resulting value is the stepsize for setting the boundaries.

Example: The maximal PD in a portfolio is 50%. In the case of five cohorts this number is divided by 5 resulting in a stepsize of 10%. So the PD intervals are:

0% - 10%, 10% - 20%, 20% - 30%, 30% - 40%, 40% - 50%.

Method 2: The boundaries are set such that every cohort has the same number of customers.

Example: For 5 cohorts 20% of the customers are in each cohort.

Method 3: The boundaries are set such that every cohort has the same number of defaults.

Example: For 5 cohorts 20% of the defaults are in each cohort.

Method 4: The boundaries are set such that the number of defaults increases linearly from the best to the worst cohort. In this case we have the following relations:

$$\xi \cdot i = A_i \quad (19)$$

$$\sum_{i=1}^k A_i = 1 \quad (20)$$

with i index of the cohort

A_i percentage of all defaults in cohort i

k number of cohorts

which are solved for ξ :

$$\xi = \frac{2}{k \cdot (k + 1)} \quad (21)$$

Once we know ξ we can use the first equation to calculate how many defaults are in each cohort and with this the boundaries can be calculated.

Example: For five cohorts ξ is equal to:

$$\xi = \frac{2}{5 \cdot (5 + 1)} = \frac{1}{15} = 0,0667$$

Consequently all defaults are distributed over the cohorts as follows:

cohort 1: 6.67% of all defaults are in this cohort

cohort 2: 13.33% of all defaults are in this cohort

cohort 3: 20% of all defaults are in this cohort

cohort 4: 26.67% of all defaults are in this cohort

cohort 5: 33.33% of all defaults are in this cohort